1

$$S = \{(1,1),(1,2),(1,3),(1,4),$$
$$(2,1),(2,2),(2,3),(2,4),$$
$$(3,1),(3,2),(3,3),(3,4),$$
$$(4,1),(4,2),(4,3),(4,4)\}$$

$$E = \{(2,4),(3,3),(4,2)\}$$

$$F = \{(2,2),(2,4),(4,2),(4,4)\}$$

$$E \cap F = \{(2,4),(4,2)\}$$

$$P(E \cap F) = \frac{2}{16} = \frac{1}{8} \approx 0.125$$

$$P(E) \cdot P(F) = \frac{3}{16} \cdot \frac{4}{16} = \frac{3}{4 \cdot 16} = \frac{3}{64} \approx 0.047$$

$\boxed{\text{Since } P(E \cap F) \neq P(E) \cdot P(F) \text{ the events are not independent}}$

(2) Let E be the event that the sum of the dice is 7. Let F be the event that the red die is 2.

$$P(F|E) = \frac{P(E \cap F)}{P(E)}$$

using: (red, green)

$$= \frac{P(\{(2,5)\})}{P(\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\})}$$

$$= \boxed{\frac{1}{6}}$$

3. See HW 3 #5

4.

(a) $P(X=-40)=P(\{(1,1)\})=\boxed{\frac{1}{16}}$

(b) $P(X=-10)$

$=P(\{(1,2),(1,3),(1,4),(2,1),(3,1),(4,1)\})=\boxed{\frac{6}{16}}=\boxed{\frac{3}{8}}$

(c) $P(X=20)$

$=P(\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4),(4,2),(4,3),(4,4)\})$

$=\boxed{\frac{9}{16}}$

(d) $E[X]=(-\$40)\left(\frac{1}{16}\right)+(-\$10)\left(\frac{6}{16}\right)+(\$20)\left(\frac{9}{16}\right)$

$=\frac{-\$40-\$60+\$180}{16}=\$\frac{80}{16}=\boxed{\$5}$

5

$A = \{(2,2), (2,4), (4,2), (4,4)\}$

$B = \{(1,2), (1,4), (2,1), (2,3), (3,2), (3,4), (4,1), (4,3)\}$

$P(A) = \frac{4}{16} = \frac{1}{4}$

$P(B) = \frac{8}{16} = \frac{1}{2}$

(a) $\dfrac{P(A)}{P(A)+P(B)} = \dfrac{1/4}{1/4 + 1/2} = \dfrac{1/4}{3/4} = \dfrac{1}{3}$

(b) $\dfrac{P(B)}{P(B)+P(A)} = \dfrac{1/2}{1/2 + 1/4} = \dfrac{1/2}{3/4} = \dfrac{2/4}{3/4} = \dfrac{2}{3}$

(c) $E[X] = (-\$5)\left(\frac{1}{3}\right) + (\$10)\left(\frac{2}{3}\right)$

$= \dfrac{-\$5 + \$20}{3} = \boxed{\$5}$